\[ \kappa_0(x_0) = K_0(x_0) + \varepsilon K_1(x_0, x_0) \]

\[ \kappa_0(x) + (x - x_0) \frac{\partial \kappa_0}{\partial x} \bigg|_{x=x_0} = 0 \]

\[ \kappa(x) = \kappa_0(x) + \varepsilon K_1(x_0, x) + \varepsilon \frac{\partial \kappa_0}{\partial \phi_0} \frac{\partial \kappa_0}{\partial \phi} \bigg|_{\phi_0=x_0} \bigg|_{\phi=x} \]

\[ H_1(x) = K_1(x_0, x) + \varepsilon \frac{\partial \kappa_0}{\partial \phi_0} \frac{\partial \kappa_0}{\partial \phi} \bigg|_{\phi_0=x_0} \bigg|_{\phi=x} \]

Both \( S_1 \) and \( H_1 \) are unknown.

The condition is that \( H_1(x) \) does not depend on \( \phi_0 \). We now average over \( \phi_0 \).

\[ H_1(x) = \frac{1}{2\pi} \int d\phi_0 K_1(x_0, x) + \frac{1}{2\pi} \int d\phi_0 \frac{\partial \kappa_0}{\partial \phi_0} \frac{\partial \kappa_0}{\partial \phi} \bigg|_{\phi_0=x_0} \bigg|_{\phi=x} \]

Integrate over \( \phi_0 \) at fixed \( x \).

\[ S_0 = 0 + \varepsilon \frac{\partial \kappa_0}{\partial \phi_0} \]

At fixed \( x \) we return to the same \( S_0 \) and therefore to the same \( \frac{\partial \kappa_0}{\partial \phi_0} \).

After \( \phi_0 = \phi_0 + 2\pi \)

\[ \frac{\partial \kappa_0}{\partial \phi_0} \text{ is periodic in } \phi_0 \text{ at fixed } x \]

\[ S_1 = a\phi_0 + p(\phi_0) \]

\( p(\phi_0) \) is periodic function.