Next we look at the case \( A^2 = 1 \)

\[
[A, H] = 0
\]

Let \( H \psi = \lambda \psi \)

then \(
\langle \psi | A \psi \rangle = \langle A \psi | A \psi \rangle^* = \langle A \psi | \psi \rangle^* = \langle \psi | A \psi \rangle
\)

\( \Rightarrow \) \( \langle \psi | A \psi \rangle = 0 \)

\( \Rightarrow \psi \) and \( A \psi \) are independent

\( H A \psi = A H \psi = A \lambda \psi = \lambda A \psi \) \( (\lambda \in \mathbb{R}) \)

\( \Rightarrow \) all eigenvalues are doubly degenerate

Kramers degeneracy

16.1 Classification of random matrix ensembles

1. No anti-unitary symmetries \( H = H^* \)
   \( p(H) = e^{-\frac{1}{2} \text{Tr} H^2} \)
   Gaussian Unitary Ensemble
   GUE

2. \( A^2 = 1 \) \( [A, H] = 1 \Rightarrow H \in \mathbb{R} \), \( \lambda = H^\dagger \)
   \( \Rightarrow H = H^\dagger \)
   \( p(H) = e^{-\frac{1}{2} \text{Tr} H^2} \)
   Gaussian Orthogonal Ensemble
   GOE