Mixing systems

The concept of mixing was first introduced by Gibbs. Qualitatively, the underlying idea of mixing is the property, for any part \( A \) of the energy surface (\( \sigma(A) \neq 0 \)), of getting spread uniformly over the energy surface as \( t \to \infty \). To get an "experimental feeling" of this feature, let us imagine a shaker in which one puts 80% of orange juice, and 20% of vodka; then, one continuously stirs the shaker. If the system is mixing, one will find, as time tends to infinity, 20% of vodka in any volume of the liquid, i.e. the vodka will be uniformly mixed with the orange juice in any part of the cocktail. Of course, such a property implies for the initial volume of vodka (or of any part of the energy surface) being strongly deformed as time runs, i.e. transforming with time in a thinner and thinner filament which ultimately will be present in any volume of the liquid (or will densely cover the energy surface).

This intuitive description of mixing makes now the mathematical definition very easy to understand:

\[
\forall A, B \subset S_E, \quad \lim_{t \to \infty} \frac{\sigma(B \cap T_t A)}{\sigma(B)} = \sigma(A). \tag{III-6}
\]

Fig.III.2 - Schematic representation of the mixing property: \( B \) is any fixed area of the surface energy, and \( A \) is a surface element considered at initial time \( t_0 \), whose evolution is drawn for successive times \( t_0 < t_2 < t_3 \). As time is running, the Hamiltonian flow \( T_t \) tends to dilute the initial surface \( \sigma(A) \) uniformly in the whole energy surface. As time tends to infinity, the fraction of the test area \( B \) occupied by \( T_t A \) is equal to the fraction of the energy surface \( S_E \) initially occupied by \( A \).

A system is said to be mixing if

Let us look at the right hand side of Eq.(III-6). One can write it as:

\[
\sigma(A) = \frac{\sigma(T_t A)}{\sigma(S_E)},
\]

since \( \sigma(T_t A) = \sigma(A) \) from Liouville's theorem, and \( \sigma(S_E) = 1 \). So the r.h.s. of Eq.(III-6)