Quadrupole moment

\[ q = \frac{2}{3} \int \left( 3r^2 \cos^2 \theta - r^2 \right) r^2 \, dr \, d\theta \]

\[ = \frac{2}{3} \int \left( 41^2 \left( 3r^2 \cos^2 \theta - r^2 \right) r^2 \, dr \, d\theta \right) \]

\[ = \frac{2}{3} \int \left( 41^2 \left( 3r^2 \cos^2 \theta - 1 \right) \right) r^2 \, dr \, d\theta \]

\[ = 0 \]

\[ \Rightarrow \text{without a mixture of } l = 2 \text{ state the quadrupole moment of the deuteron would be zero.} \]

A mixture of \( l = 1 \) is not possible because parity is a good quantum number.

\[ 141^3 = [4_0 + 84^1] = [4_0] + 2[4_1 + 84^1] + [84^3] \]

\[ \Rightarrow \Omega = \frac{2}{\sqrt{3}} \gamma_1 \text{ and } \xi = \frac{1}{\sqrt{3}} \gamma_1 X^1 + S^1 X^2 \gamma_1 \]

\[ <X^0_1 | 21, 21 | X^0_1> = -1 \quad <\gamma^0_1 | \gamma^0_1> = 1 \]

\[ \text{does not contribute to the quadrupole moment} \]

Take \( M = 1 \)

\[ <\gamma_1 | S^0_1 | X^1_1> = \frac{1}{\sqrt{3}} \quad <\gamma_1 | S_0 \gamma_0 X_1 | X_1> = \]

\[ \text{are combinations of } S_0 \text{ and } S \text{ and have the spin } 1 \]

\[ \text{and } <\gamma_0 | S_0 \gamma_0 X_1 | X_1> = 2 \]