We now show that within a given unitary representation \( A^2 = \pm 1 \)

\[ A^2 \text{ is unitary} \quad \Rightarrow \quad \underline{u^*uu^*} = \underline{u^*u} = 1 \]

\[ \Rightarrow \quad \text{within a given unitary representation} \]

\[ A^2 = \lambda \quad |\lambda| = 1 \]

\[ A = uu^* = \lambda I \]

\[ \Rightarrow \quad u^{-1}uu^*u = \overline{\lambda} I \quad u = \lambda I \]

\[ \Rightarrow \quad \lambda^2 = \lambda \quad \in \mathbb{R} \quad |\lambda| = 1 \]

\[ \Rightarrow \quad \lambda = \pm 1 \]

\[ \Rightarrow \quad 3 \text{ only possibilities:} \]

\[ i) \text{ no anti-unitary symmetry} \]

\[ ii) \quad A^2 = 1 \]

\[ iii) \quad A^2 = -1 \]

Consequences of anti-unitary symmetry:

If \( A^2 = 1 \), it is always possible to find a basis for which the Hamiltonian is real.

\[ [H, A] = 0 \quad \Rightarrow \quad H^* = H \]

\[ A^2 = 1 \]