how take \( R_k \rightarrow 0 \) and \( L_k = k \)

\[
\begin{align*}
\theta_1 & = \frac{1}{k^2} \cot \delta_1 - \frac{1}{2}
\end{align*}
\]

\[
\theta_2 \cot \delta = -\frac{1}{k} + \frac{1}{2} k^2 \int_{0}^{\infty} \phi^2(r) \, dr
\]

\[
\int_{0}^{\infty} (\phi \phi_2 - u u_1) \, dr
\]

\( \phi_k \) differs from \( u \) only for \( k < R \)

Here, the wave function can be replaced by the wave function at zero.

\[
\int_{0}^{\infty} \phi^2(r) \, dr
\]

\[
\int_{0}^{\infty} \phi^2(r) \, dr
\]

\[
\Rightarrow \quad \theta \cot \delta = -\frac{1}{k} + \frac{1}{2} R_0 k^2 + \cdots
\]

\( R_0 \) is known as the effective range.

This is an expansion in powers of \( R_0 k \) and is known as the effective range expansion.

This expansion can also be extended to higher partial waves.