For the 2nd order term we obtain
\[ M_2 = \frac{9\pi}{2} \cdot \frac{1}{4} \int_0^\infty \frac{r^2 \sin^2 r}{r^2} \left( (2\pi r)^2 + 2 \sin^2 r \right) \, dr \]

For the fourth order term we obtain
\[ M_4 = \frac{9\pi}{2} \cdot \frac{9}{16} \int_0^\infty \frac{r^4 \sin^2 r}{r^2} \left( \frac{\sin^2 r}{r^2} + 2(2\pi r)^2 \right) \, dr \]

Solution can be determined numerically by minimizing w.r.t. \( F \).

\[ \varepsilon^2 = 20.0055 \quad \Rightarrow \quad M_\pi = 1425 \text{ MeV} \]

\[ F_\pi = 93 \text{ MeV} \]

6c) Nucleon-\( \sigma \) mass splitting

The chiral-\( \sigma \) mechanism is a classical solution. Intuitively we expect that the rotational energy is given by \( \frac{J^2}{2I^2} \) moment of inertia.

We thus find \( M^\pi = M_0 + \frac{1}{2} \left( \frac{J^2}{2I} \right) \)

\[ M_0 = M_0 + \frac{3}{2} \cdot \frac{\sigma}{2I} \]

\[ = \frac{3}{2} \cdot M_0 - \frac{3}{2} \cdot \frac{\sigma}{2I} \]

\[ = \frac{3}{2} \cdot M_0 - M_\pi = \frac{3}{2} \cdot \text{GeV} \approx 50 \text{ MeV} \]

Let us now do a more sophisticated treatment which also shows that \( I = J \).