\[ \nabla^2 \phi_a - (\frac{\partial}{\partial x^2} \phi_a) \phi_a = 0 \]

This equation has finite energy solution

Energy is given by \( E = \frac{1}{2} \int (\partial_a \phi^a)^2 d^2x \)

\[ \Rightarrow \phi(x) \xrightarrow{\text{constant}} \phi_0 \]

\[ \Rightarrow \text{Field is a mapping from sphere } \rightarrow \text{ sphere} \]

\( \hat{\phi} = 1 \) in a sphere surface

These mapping can be classified by their winding number.

Mathematically \( \pi_2(S^2) = \mathbb{Z} \)

Mappings with different winding number cannot be obtained from each other by continuous deformations.

5a) Intermezzo \( \pi_1(S^1) = \mathbb{Z} \)

Map \( \lambda_1(\theta) = 0 \) cannot be deformed to \( \lambda_1(\theta) = 0 \) by continuous deformations. To show this we introduce the winding number.