then \( W(\phi_a) = \lambda^{2-d} V_1(\phi_a) + \lambda^d V_2(\phi_a) \)

This means that if we would have a static solution, we could lower its action by rescaling for \( 2 - d < 0 \).

Only possible solution \( V_1(\phi) = 0 \Rightarrow \phi = \text{const.} \)
\( V_2(\phi) = 0 \Rightarrow \phi \text{ is zero} \)

So we have only trivial solutions.

However, solitons can exist in 3 dimensions.
This can happen when solitons cannot decay because of topological obstruction.

5c) Non-linear O(\( N \)) model in \( d = 2 \)

Lagrangian \( L = \frac{1}{2} (\partial_a \phi)^2 \) with \( \phi^2 = 1 \)
\( a = 1, \ldots, N \)

We will take \( N = 3 \)

Constraint can be included by Lagrange constraint

\( L \rightarrow \frac{1}{2} (\partial_a \phi)^2 + \lambda (\phi^2 - 1) \)

\( EL \quad \nabla^2 \phi_a + \lambda \phi_a = 0 \)

Take scalar product with \( \phi_a \Rightarrow \phi_a \nabla^2 \phi_a + \lambda = 0 \)