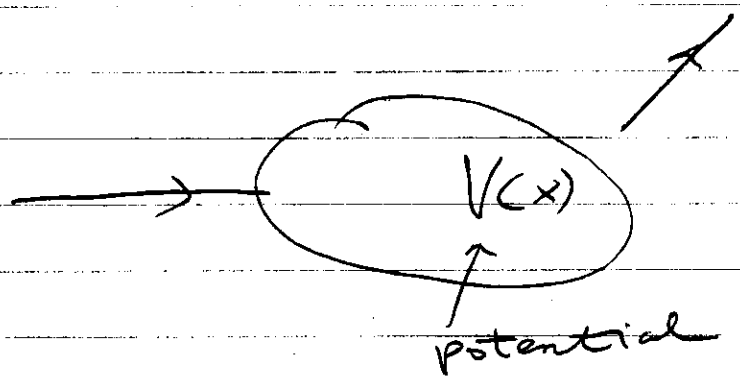
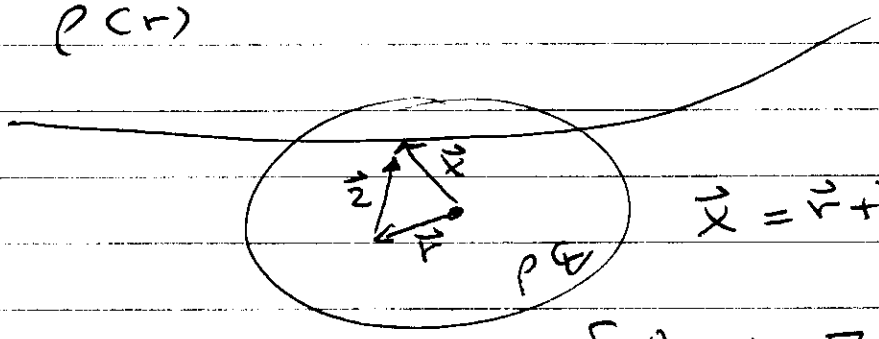
1f) Scattering in quantum mechanics



First born approximation $\frac{d\sigma}{d\Omega} = |f(q)|^2$

$$f(q) = -\frac{m}{2\pi\hbar^2}\int V(x)\, e^{\frac{iqx}{\hbar}}\, d^3x$$

assume that potential is due to a charge density $\rho(r)$



$\vec{x} = \vec{r} + \vec{z}$

$$\int d^3r\,\rho(r) = Z_2 e$$

$$V(x) = z_1 z_2 e^2 \int d^3r \frac{\rho(r)}{z} e^{-\frac{z}{a}}$$ ← shielding factor

$$\Rightarrow f(q^2) = \frac{m z_1 e}{2\pi\hbar^2}\int \underbrace{d^3x}_{d^3z}\, e^{\frac{iq(\vec{r}+\vec{z})}{\hbar}} \int d^3r\, \frac{\rho(r)}{z} e^{-\frac{z}{a}}$$

$$= \frac{m z_1 e}{2\pi\hbar^2}\int d^3z\, \frac{e^{\frac{iq\vec{z}}{\hbar}} e^{-\frac{z}{a}}}{z} \int d^3r\, \rho(r) e^{\frac{iqr}{\hbar}}$$

$$= \left.\frac{d\sigma}{d\Omega}\right|_{\substack{\text{rutherford}\\ \text{Mott}}} \int d^3r \rho(r)\, e^{\frac{iqr}{\hbar}}$$