4. Continuous symmetry

a) Goldstone's theorem

If a continuous symmetry is spontaneously broken, the transverse correlations show a power law behavior for $T < T_c$

Marmin–Wagner theorem (Coleman theorem)

Long range order is absent for $d \leq 2$

Exception: Kosterlitz–Thouless theorem in the classical XY model

5) Goldstone's theorem for $O(n)$ model

$$ H = \int d^4 r \left( \frac{1}{2} \left( \delta \mathbf{s} \right)^2 + \frac{1}{2} \mathbf{r} \cdot \mathbf{s} + \frac{1}{4} n s^2 \right) - \frac{n}{2} s $$

$$ s^2 = \sum_{\alpha=1}^{n} \frac{\mathbf{s} \cdot \mathbf{s} (r)}{n} \mathbf{s} (r) = \sum_{\alpha \neq \beta} (\mathbf{s} \mathbf{s}) $$

$$ Z = \int \prod_{r} d s (r) \ e^{-\beta H} $$

For $n = 0$, $H$ has an $O(n)$ symmetry.

Invariance: $s \rightarrow 0 \ s \quad O(\lambda)$