Goldstone degrees of freedom

\[ \Sigma = (\mathbf{U}^+ \mathbf{V})_{f_g} \]

transforms \( \Sigma \rightarrow U_R^+ \Sigma U_L \)

QCD is invariant under vector transformation

= Goldstone bosons related by a vector transformation have the same properties

= Goldstone bosons appear in multiplets

\[ \Sigma_{f_g} \rightarrow U_{f_f}^* \Sigma_{f_f g_l} U_{g_l} \]

\[ = U_{g_l}^* U_{f_f}^* \Sigma_{f_f g_l} \]

\( \Sigma \) is a representation of \( \mathbf{U}_N(N_f) \)

irreducible representations: singlet \( \Sigma_{f_g} \)

\[ U_{g_l}^* U_{f_f}^* \delta_{f_f g_l} = \delta_{f_g} \]

For \( N_f = 3 \) we have \( 3^* \times 3 = 10 + 8 \)

The singlet state appears to be not a true goldstone boson. The reason is the explicit breaking of \( U(3) \) by topological