one "crosses" a prime. The first 25 steps of this irregular staircase are shown on Fig.I.1. Has this staircase function an average behaviour? Gauss observed as early as 1792 that the density of prime numbers $d\pi(x)/dx$ appears on the average to be $1/\ln x$.

![Plot of the function $\pi(x)$ for $1 < x < 100$](image)

Fig.I.1 - Plot of the function $\pi(x)$ for $1 < x < 100$

He was thus led, from "empirical observation" of primes, to approximate $\pi(x)$ by the integral logarithm $Li(x)$

$$
\pi(x) \approx Li(x) = \lim_{\varepsilon \to 0} \left[ \int_0^{1-\varepsilon} \frac{dt}{\ln t} + \int_{1+\varepsilon}^x \frac{dt}{\ln t} \right]
\approx Li(2) + \int_2^x \frac{dt}{\ln t} = 1.04 + \int_2^x \frac{dt}{\ln t}.
$$  \hspace{1cm} (I-3)

$Li(x)$ admits the following expansion

$$
Li(x) = \gamma + \ln \ln x + \sum_{n>1} \frac{(\ln x)^n}{n \cdot n!}.
$$  \hspace{1cm} (I-4)