\[ m_1(L) = \bar{n}(L) = \sum_{k=0}^{\infty} k E(k; L) = y_1(L) = L \]  

(II-32)

\[ \Sigma^2(L) = m_2(L) = (\bar{n}(L) - L)^2 = \sum_{k=0}^{\infty} (kL)^2 E(k; L) = y_2(L) - y_4(L) \]  

(II-32')

\[ \gamma_1(L) = \left\{ \sum_{k=0}^{\infty} (kL)^3 E(k; L) \right\} / m_2(L) = (y_3(L) - 3y_2(L) + 2y_4(L)) / m_2^2(L) \]  

(II-32'')

\[ \gamma_2(L) = \left\{ \sum_{k=0}^{\infty} (kL)^4 E(k; L) - 3m_2^2(L) \right\} / m_2^2(L) \]  

(II-32''')

\[ = (y_4(L) - 7y_2(L) + 6y_3(L) - 6y_4(L)) / m_2^2(L) \]

Thus, \( \gamma_1 \) and \( \gamma_2 \) are given in terms of the functions \( E(k; L) \) and all values of \( k \) appear. However, they are in fact (2)-, (2+3)- and (2+3+4)- point measures respectively, as can be seen from the last equalities in (II-32). When dealing with the 2-level cluster function \( \gamma_2(x_1, x_2) \) one uses the notation \( \gamma_2(x) \). \( \gamma_2 \) is related to the spacing distributions by

\[ 1 - \gamma_2(x) = \sum_{k=0}^{\infty} p(k; x) \]  

(II-33)

and \((1 - \gamma_2(x)) \) gives the probability of observing a level in an infinitesimal interval \( dx \) at a distance \( x \) from a given level. An alternative form of (II-32') is

\[ \Sigma^2(L) = L - \int_0^L (L - r) \gamma_2(r) dr \]  

(II-34)

Finally, consider the least-square statistic \( \Delta_3(L) \) introduced in the previous Section (I-38). It can be shown that its ensemble average \( \overline{\Delta_3(L)} \) can be obtained as follows

\[ \overline{\Delta_3(L)} = (2/L^4) \int_0^L \left( L^2 - 2L^2r + r^2 \right) \Sigma^2(r) dr \]  

(II-35)

Therefore, like \( \Sigma^2(L) \), it is also a 2-point measure (some particular integral of \( \gamma_2 \)).

\(^{(k)}\) Take as coordinates the center of the interval \( x = (x_1 + x_2) / 2 \) and the relative coordinate \( x = x_1 - x_2 \). Then \( \gamma_2(x_1, x_2) = \gamma_2(x, x) \) but as \( \gamma_2 \) does not depend on \( \chi \), one simply writes \( \gamma_2(x) \).