equal to the spectral averaged density \( \langle p(E) \rangle_s \). The mapping \( E \rightarrow \chi \) (Eq. (II-22')) (unfolding) is such that the ensemble-averaged local spacing \( 1/\rho(\chi) \) is stationary (independent of \( \chi \)) and equal to unity. And the k-level cluster functions \( \gamma_k \), functions of the variables \( x_j \), are also stationary (they only depend on the relative coordinates \( x_{ij} = x_i - x_j \)). This means that the fluctuation properties of several segments of a spectrum located at different positions will be the same: from the point of view of fluctuations, the spectrum is translationally invariant. Furthermore, after unfolding, a spectral average is equal to an ensemble average.

An alternative way to characterize fluctuations consists to deal with spacing distributions\(^(*)\) and related quantities. In (II-21), instead of integrating from \(-\infty\) to \(+\infty\) without any restriction, one integrates some of the variables outside the interval \([\alpha, \alpha + L]\) whereas the others are integrated inside it. Assume that the unfolding (mapping \( E \rightarrow \chi \)) has been performed. One defines (\( N \gg k \gg 0 \))

\[
E(k; L) = \lim_{N \to \infty} \frac{N!}{(N-k)!} \int_{\chi_1}^\chi_2 \cdots \int_{\chi_n}^\chi_{n+k} \cdots \int_{\chi_{n-k}}^\chi_{n} p_n(x_i, \ldots, x_n). \tag{II-26}
\]

If the system is stationary, \( E(k; L) \) will be independent of \( \alpha \). \( E(k; L) \) is the probability that in a sequence \( \{x_i\} \) of levels with mean spacing unity an interval of length \( L \) taken at random contains exactly \( k \) levels. One useful aspect of the functions \( E(k; L) \) is that they are very directly connected to the spacing distributions [MdC-72]

\[
p(k; L) = \left( \frac{d^2}{dx^2} \right) \sum_{j=0}^{k} (k-j+1) E(j; L) \tag{II-27}
\]

In particular, for the nearest-neighbour spacing distribution \( p(x) \)

\[
p(x) = p(0; x) = \left( \frac{d^2}{dx^2} \right) E(0; x). \tag{II-27'}
\]

The probability \( F(k; L) \) that in a sequence \( \{x_i\} \) of levels with mean spacing unity, an interval \([x_{\alpha}, x_{\alpha} + L]\) of length \( L \) which starts at a level \( x_{\alpha} \) contains exactly \( k \) levels is also given in terms of the functions \( E(k; L) \):

\(^(*)\) The distribution of nearest-neighbour spacings has been denoted, and will continue to be when no confusion is possible, by \( p(x) \equiv p(0; x) \); \( p(k; x) \) denotes the distribution of spacings \( S = x_{i+k+1} - x_i \) between two levels \( x_i \) and \( x_{i+k+1} \) having \( k \) levels in-between (\( k = 0, 1, 2, \ldots \)).