1. The ensemble is invariant under every orthogonal transformation

\[ H' = W^T H W \]  

where \( W \) is any real orthogonal matrix, i.e., the probability \( \mathcal{P}(H) dH \) that a matrix \( H \) will be in the volume element \( dH \) (Eq. (II-6)) is invariant under orthogonal transformations(*)

\[ \mathcal{P}(H') dH' = \mathcal{P}(H) dH \]  

(II-10)

2. The various elements \( H_{ij} (i \neq j) \) are independent random variables.

We make the first requirement because we don't want that any given state plays a particular role: all basis states, and therefore all states, should behave in the same way. The second requirement has no special physical origin. It is put for the sake of simplicity with the hope of leading to a mathematically soluble problem.

Similarly, the Gaussian Unitary Ensemble (GUE) in the space of Hermitian matrices is defined by the properties

1'. The ensemble is invariant under every unitary transformation

\[ H' = U^+ H U \]  

where \( U \) is any unitary matrix, i.e., the probability \( \mathcal{P}(H) dH \) that a matrix \( H \) will be in the volume element \( dH \) (Eq. (II-8)) is invariant under unitary transformations

\[ \mathcal{P}(H') dH' = \mathcal{P}(H) dH \]  

(II-12)

2'. The various elements \( H_{ij} (i \leq j) \), \( H_{ij} (i < j) \) are independent random variables, i.e., \( \mathcal{P}(H) \) is a product of \( N^2 \) functions.

These two requirements (1. and 2. or 1'. and 2') determine uniquely the ensembles. The function \( \mathcal{P}(H) \), which will also be invariant under the corresponding automorphism, can be written

\[ \mathcal{P}_{NP}(H) = K_{NP} \exp \left\{ -\frac{\text{Tr}(H^2)}{4\sigma^2} \right\} \]  

(II-13)

(*) Notice that from (II-10) and from the invariance of the measure \( dH \), one has that \( \mathcal{P}(H) \) must also be invariant under orthogonal transformations.