where \( \rho_{av}(s) \) is the average density of \( \{ s_i \} \). For instance, consider the (fluctuation-free) sequence \( s_k = k^2 \) \( (k = 1, 2, \ldots) \); then \( N_{av}(s) = \sqrt{s} \) and \( x_k = k \), a sequence of equally spaced points or picket fence. In summary, after unfolding, we shall study quantities related to
\[
\hat{N}(x) = \hat{N}_{av}(x) + \hat{N}_{fl}(x) = x + \hat{N}_{fl}(x).
\] (I-29)

When considering fluctuation properties of sequences \( \{ x_i \} \) we shall come across different situations: i) cases in which the system is known to be, from a statistical point of view, translational invariant or stationary, i.e., the fluctuation properties are the same irrespective of which region of the spectrum (of the sequence) one is considering; ii) cases in which the system is not stationary but one is interested in asymptotic properties of the spectrum.

The question now is to discover the stochastic laws governing sequences having very different origins, as illustrated on Fig.1.8, which is inspired from a similar figure of Ref.[BFF-81]. There are displayed six spectra, each containing 50 levels. Column (a) corresponds to a Poisson system: Take a random variable \( s \) whose probability density \( p(x) \) is \( e^{-x} \). Construct a sequence \( \{ s_i \} \)
\[
x_i = 0, \quad x_{i+1} = x_i + s_i \quad i = 1, 2, 3, \ldots
\] (I-30)
where \( s_i \) are outcomes of independent trials of the variable \( s \). The resulting spectrum is what is called a Poisson spectrum, which is obviously stationary. For instance, if one studies the counting rate of a decaying source, the successive times of decay \( x_i \) will form a Poisson spectrum, the time being measured in units of the mean life of the source. Column (b) shows an example of a segment of prime numbers in the interval [7791097-7791877] Ref.[Si-79]; column (c) the resonance energies \( J^\pi = 1/2^+ \) of the compound nucleus observed in the reaction \( n + {}^{166}\text{Er} \) (see Section II); column (d) the eigenvalues (associated to eigenfunctions with a given symmetry) corresponding to the transverse vibrations of a membrane whose boundary is the Sinai's billiard (see Section IV); column (e) the positive imaginary part of the 1551-th to the 1600-th zero of the Riemann zeta function [HM-63]; column (f) an equally spaced sequence of levels (picket fence). Columns (a) and (f) represent two limiting cases, maximum randomness and no randomness at all respectively.

Can one deduce some features just by inspecting Fig.1.8? Arrows indicate spacings \( s_i = x_{i+1} - x_i \) which are smaller than 1/4. The Poisson spectrum shows 12 arrows out of 49 spacings, the prime number "spectrum" shows 9 arrows, the \( \text{Er} \) spectrum only 2 arrows, the frequencies of the membrane 3 arrows, the zeros of \( \xi(s) \) no arrow and, of course, the picket fence no arrow. One therefore sees a statistical

\(^*\) The spectra have been rescaled to the same spectrum span [0,49], thereby introducing an artificial rigidity (see below).