Proceeds from the principal should be used to invite eminent scientists to lecture in Göttingen. Lorentz gave some lectures under the general title "Alte und neue Fragen der Physik". At the end of one of them he asked "In an enclosure with a perfectly reflecting surface there can form standing electromagnetic waves... The mathematical problem is to prove that the number of sufficiently high overtones which lie between \( \nu \) and \( \nu + d\nu \) is independent of the shape of the enclosure and is simply proportional to its volume". If one believes an apocryphal report Hilbert predicted that the theorem would not be proved during his life. Less than two years later Hermann Weyl, who was present at the Lorentz's lecture, using the theory of integral equations which his teacher Hilbert developed only a few years before, proved the theorem, long before his death.

Let us now go back to the initial two-dimensional problem. Progress has been made since the pioneering work by Weyl. We are interested in extracting a smoothed eigenvalue distribution \( N_{\text{av}}(E) \), i.e. the smoothed function giving the number of eigenvalues less than or equal to \( E \) in order to study the fluctuations or oscillations of the exact eigenvalue distribution around the averaged value \( N(E) \). In the context of the previous example on prime numbers, we are searching the function \( N_{\text{av}}(E) \) which has a similar relationship to \( N(E) \) as \( \rho(x) \) to \( \pi(x) \). Reference [BH-76] gives a complete account of the results obtained so far in this field. The function \( N_{\text{av}}(E) \) can be written

\[
N_{\text{av}}(E) = \frac{\sigma}{4\pi} \left( \frac{E}{4\pi} \right)^{1/2} \sqrt{E} + K + O \left( E^{-\eta/2} \zeta(n \sqrt{E}) \right),
\]

where \( 0 < \eta < 1 \). In (1.24) \( \sigma \) is the surface of the area \( \Omega \) (Weyl's term) and \( \gamma \) is the perimeter of the boundary \( \Gamma \). \( K \) is a constant term containing complex information on the geometrical and topological properties of the domain. The geometrical features contributing to the constant term are: i) Curvature contribution

\[
\left( \frac{1}{4\pi} \right) \int_{\Gamma} \kappa(\ell) \, d\ell,
\]

where \( \kappa(\ell) \) denotes the local curvature; for instance, the curvature contribution for the circle is 1/6. ii) Corners contribution; for a square (or a rectangle), it is \( 4 \times (1/48) \). The topological features concern the connectivity of the surface; for a multiply connected drum containing \( r \) holes, the contribution to the constant term is \( (1-r) \times (1/6) \).

On Fig.1.7 are compared the exact function \( N(E) \) and the smoothed function \( N_{\text{av}}(E) \) given by Eq.(1.24) for two different shapes, namely a quarter of a circle and a stadium (see Section III). It can be seen that \( N(E) \) indeed reproduces perfectly the average behaviour of \( N_{\text{av}}(E) \), not only asymptotically but starting from the bottom of the spectrum.

(*) If one uses Neumann instead of Dirichlet boundary conditions, (1.24) is still valid except for the sign of the perimeter term.