1. Consider the integral

\[ I = \int_{0}^{\infty} g(x)dx \log(1 + e^{\alpha - \beta x}) \]  

(1)

with \( g(x) \) the level density that is slowly varying at the Fermi level.

a) Derive the low temperature expansion of this integral to order \( 1/\beta \).

b) Find numerically the parameter range of \( \alpha \) and \( \beta \) for which this expansion is more accurate than one percent. Do this by drawing a graph in the \( \alpha - \beta \) plane.

2. If the potential is nonzero only for \( r < R_c \) and the reduced wave function for large \( r \) is given by

\[ u_0(r) = e^{-ikr} - \eta_0 e^{ikr}, \]  

(2)

show that the elastic cross-section is given by

\[ \sigma^{el} = \frac{\pi}{k^2} |1 - \eta_0|^2. \]  

(3)

3. Consider a particle in one dimension with potential

\[ V(x) = x^4. \]  

(4)

a) Determine the semi-classical integrated density of states and find the \( \hbar \) dependence of the eigenvalues of the Schrödinger equation.

b) Use scaling arguments in the Schrödinger equation to derive an exact result for the \( \hbar \) dependence of the eigenvalues. Your result should agree with what you have found in a).