Homework Set # 4, due March 10, 2008

1. Show that the magnetic moment of an electron moving in a circular orbit is given by

\[ \mu = -\frac{\hbar e}{2mcL}, \]  

with \( L \) the orbital angular momentum. Assume that the charge and the mass of the electron is distributed uniformly along the orbit.

2. In this problem we calculate the magnetic moment of the proton and the neutron in a naive quark model. Experimentally the values are \( \mu_p = 2.790\text{n.m.} \) and \( \mu_n = -1.93\text{n.m.} \), where \( \text{n.m.} \) is the nuclear magneton: \( \hbar e/2M_Nc \). Below we use units with \( \hbar = c = 1 \).

a) Show that the spin part of the wave function is given by

\[ \psi(\frac{1}{2}, \frac{1}{2}) = \sqrt{2/3}\chi(1, 1)\phi(\frac{1}{2}, -\frac{1}{2}) - \sqrt{1/3}\chi(1, 0)\phi(\frac{1}{2}, \frac{1}{2}), \]  

where \( \chi \) is the triplet wavefunction of the two quarks of the same flavor, and \( \phi \) is the spinor of the remaining quark. Hint: Use that the wave function is normalized and that \( J_+\psi(\frac{1}{2}, \frac{1}{2}) = 0 \).

b) Write down the spin part of the wave function for the neutron and the proton.

c) Show that the magnetic moments are given by

\[ \mu_p = \frac{4}{3}\mu_u - \frac{1}{3}\mu_d, \quad \mu_p = \frac{4}{3}\mu_d - \frac{1}{3}\mu_u \]  

with \( \mu_u \) and \( \mu_d \) the magnetic moments of the up and down quarks (i.e. \( \mu_q = 2e_q/m_q \)). Use that the magnetic moment operator is given by

\[ \sum_q \frac{e_q}{2m_q} \sigma_q \]  

with \( \sigma_q \) the z-component of the spin \( \frac{1}{2} \) operator for quark \( q \).

d) Calculate the magnetic moment of the neutron and the proton using that \( m_q = m_N/3 \) for both the u-quark and the d-quark and compare with the experimental result.
3. In this problem we consider an MIT bag model, where the bag is given by a cube of length $L$. Find the energy of the ground state of a Dirac particle in this cube. On dimensional grounds the result should be of the form $E_0 = \alpha_c/L$. Compare $\alpha_c$ with the value of $\alpha$ for a spherical bag where, as discussed in the lecture, the ground state energy is given by $E_0 = \alpha/R$. 