Poincare section

2 dof \( x_1, x_2, \dot{x}_1, \dot{x}_2 \)

\( E \) conserved \( \Rightarrow \) trajectory in 2d

Poincare section is the collection of section points with \( \dot{x}_2 > 0 \)

2 conditions on 4 variables \( E, x_1 = 0 \)

\( \Rightarrow \) points cover a 2d area

For two conserved quantities on \( x_1, x_2, \dot{x}_1, \dot{x}_2 \)

with \( x_1 = 0 \) we have 3 conditions on 4 variables \( \Rightarrow \) section points are on a curve.

Example

\( h.o. \) in 2d

2 conserved quantities

\[ \frac{1}{2} \dot{x}_1^2 + \omega_1 x_1^2 = E_1 \]

\[ \frac{1}{2} \dot{x}_2^2 + \omega_2 x_2^2 = E_2 \]

If \( \frac{\omega_1}{\omega_2} \in \mathbb{Q} \) then we have only a finite number of section points.