\[ \frac{\partial y_k}{\partial x_j} \frac{\partial y_q}{\partial x_i} = \delta_{kq} \]

\[ \frac{\partial Q_k}{\partial q_i} \frac{\partial P_q}{\partial p_j} - \frac{\partial Q_k}{\partial p_j} \frac{\partial P_q}{\partial q_i} = \delta_{kq} \]

If the Poisson brackets are invariant, then the transformation is canonical.

\( d\Pi = dq_1 \ldots dq_n \, dp_1 \ldots dp_n \)

Volume: \( d\Pi = dq_1 \ldots dq_n \, dp_1 \ldots dp_n \)

Canonical transformation:

\( d\Pi' = dq_1 \ldots dq_n \, dp_1 \ldots dp_n \)

**Liouville's theorem**: \( d\Pi = d\Pi' \)

To prove this, we have to show that the Jacobian \( (q, p) \rightarrow (q', p') \) is equal to 1.