Finally we choose for the $\delta e$

$$Q_i \ldots Q_n P_i \ldots P_n$$

$$\Delta i = Q_i, \Delta j = P_j \Rightarrow \delta e = 0$$

$$0 = \sum \left( [Q_i, Q_j] [Q_i, P_j] + [P_i, Q_j] [Q_i, P_j] \right)$$

$$= [P_i, P_j] = 0$$

Interchange $P_j \leftrightarrow Q_i \Rightarrow [Q_i, Q_j] = 0$

Finally we choose $\Delta i = Q_i, \Delta j = Q_j$

$$\Rightarrow \sum \left( [Q_i, Q_j] [Q_i, Q_j] + [P_i, Q_j] [Q_i, Q_j] \right)$$

$$= [Q_i, Q_j] [Q_i, Q_j] = \delta i j$$

$$\Rightarrow [P_i, Q_j] [P_i, Q_j] = \delta i j$$

$$\Rightarrow [P_i, Q_j] = -\delta i j$$

Invariance of the Poisson bracket for an arbitrary function of $P$ and $Q$ then follows by induction.