Some properties of Poisson brackets

\[ [f, g] = - [g, f] \]
\[ [f, c] = 0 \]
\[ [f_1 + f_2, g] = [f_1, g] + [f_2, g] \]
\[ [f_1, f_2, g] = f_1 [f_2, g] + f_2 [f_1, g] \]
\[ \{f, g_i \} = -\frac{\partial f}{\partial x_i} \]
\[ [p_i, p_k] = 0 \]
\[ [q_i, q_j] = 0 \]
\[ [p_i, q_k] = 0 \]
\[ [p_i, p_k] = -\delta_{ik} \]

Jacobi identity

\[ [f, [g, h]] + [g, [h, f]] + [h, [f, g]] = 0 \]

\[ [f, [g, h]] = \frac{\partial f}{\partial q_k} \left( \frac{\partial g}{\partial q_i} \frac{\partial h}{\partial p_k} - \frac{\partial g}{\partial p_k} \frac{\partial h}{\partial q_i} \right) - \frac{\partial f}{\partial p_k} \left( \frac{\partial g}{\partial q_i} \frac{\partial h}{\partial q_k} - \frac{\partial g}{\partial p_k} \frac{\partial h}{\partial p_i} \right) + \text{cyclic} \]

\[ = \frac{\partial f}{\partial q_k} \left( \frac{\partial g}{\partial q_i} \frac{\partial h}{\partial p_k} - \frac{\partial g}{\partial p_k} \frac{\partial h}{\partial q_i} \right) - \frac{\partial f}{\partial p_k} \left( \frac{\partial g}{\partial q_i} \frac{\partial h}{\partial q_k} - \frac{\partial g}{\partial p_k} \frac{\partial h}{\partial p_i} \right) + \frac{\partial f}{\partial p_k} \frac{\partial^2 h}{\partial q_i \partial q_k} + \text{cyclic} = 0 \]