Poisson brackets (section 0.5)

Let us consider a function on phase space $f(p, q, t)$

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial p} \dot{p} + \frac{\partial f}{\partial q} \dot{q}$$

$$\frac{\partial H}{\partial p_k} - \frac{\partial H}{\partial q_k} = 0$$

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial p} \dot{p} + \frac{\partial f}{\partial q} \dot{q}$$

$$[f, H] = \frac{1}{2} \left( \frac{\partial f}{\partial p} \frac{\partial H}{\partial q} - \frac{\partial f}{\partial q} \frac{\partial H}{\partial p} \right)$$

Poisson bracket

This is the classical limit of a commutator.

If $f$ is an integral of motion then $\frac{df}{dt} = 0$ then $\frac{\partial f}{\partial t} + [f, H] = 0$.

If $\frac{df}{dt} = 0$ as well then $[H, f] = 0$.

Generalized Poisson bracket

$$[g, f] = \sum_{k} \left[ \frac{\partial f}{\partial p_k} \frac{\partial g}{\partial q_k} - \frac{\partial f}{\partial q_k} \frac{\partial g}{\partial p_k} \right]$$