Example 2

Moving and fixed system have common origin.

\[ I_1, I_2, I_3 \text{ moment of inertia w.r.t cm} \]

\[ \alpha = (0, 0, \ell) \]

x3 axis is principal axis, choose \( \psi = 0 \) (does not mean that \( \phi = 0 \))

Near moment of inertia w.r.t \( O \)

\[ I_1 \to I_1 + \ell^2 \]

\[ I_2 \to I_2 + \ell^2 \quad \text{(choose } \psi = 0 \text{ due to symmetry)} \]

\[ \Rightarrow \quad L = \frac{1}{2} (I_1 + \ell^2) \left( \dot{\psi}^2 + \dot{\phi}^2 \sin^2 \theta \right) \left( \ell^2 \right) \]

\[ + \frac{1}{2} I_2 \left( \dot{\psi}^2 + \dot{\phi} \cos \theta \right)^2 - mg \ell \cos \theta \]

\( \psi \) and \( \phi \) are cyclic \( \Rightarrow \) \( p_\psi \), \( p_\phi \) are conserved

\[ p_\psi = \frac{\partial L}{\partial \dot{\psi}} = I_1 \left( \dot{\psi}^2 + \dot{\phi} \cos \theta \right) = I_2 \omega_3 = M_3 \]

\[ p_\phi = \frac{\partial L}{\partial \dot{\phi}} = \frac{(I_1 + \ell^2) \sin^2 \theta \dot{\phi} + I_2 \left( \dot{\psi} + \dot{\phi} \cos \theta \right) \cos \theta}{M_3 \sin \theta} \]

\[ = M_3 \ell \cos \theta \quad \text{conserved} \]