3 a) Motion in 1d

Cartesian coordinates \( L = \frac{1}{2} m \dot{x}^2 - u(x) \)

\[
\Rightarrow \frac{dx}{dt} = \sqrt{2m(E-U)} \quad E = \frac{1}{2} m \dot{x}^2 + U
\]

\[
= \int \frac{dx}{\sqrt{2m(E-U)}}
\]

Eq. of motion in 1d can always be integrated.

2 turning points =
motion is bounded for such type potential

3 b) Reduced mass

\[
L = \frac{1}{2} m_1 \dot{r}_1^2 + \frac{1}{2} m_2 \dot{r}_2^2 - U(r_1 - r_2)
\]

\[
R = \frac{m_1 \dot{r}_1 + m_2 \dot{r}_2}{m_1 + m_2} \quad r = r_1 - r_2
\]

\[
= \frac{r_1}{m_1 + m_2} \quad r_2 = -\frac{m_1 \dot{r}_1}{m_1 + m_2}
\]

\[
L = \frac{1}{2} m_1 \dot{r}_1^2 + \frac{1}{2} m_2 \dot{r}_2^2 - U(r_1 - r_2)
\]

\[
= \frac{1}{2} \lambda \dot{r}^2 - U(r)
\]

Reduced mass \( \lambda = \frac{m_1 m_2}{m_1 + m_2} \)