Galileo's Relativity Principle

It is always possible to find coordinates such that time and space are homogeneous and space is isotropic. Such frame is called an inertial frame.

This implies that a free particle has the same equations of motion all over space-time.

\[ L_{\text{free}} = L \left( u^2 \right) \]

The Lagrangian of a free particle does not depend on \( q_{\mu} \) and \( t \). Because space is isotropic it can only depend on \( u^2 \).

\[ \frac{\partial L}{\partial u^2} = \text{constant} \Rightarrow \dot{u} = \text{constant} \]

Lagrangian equation of motion \( \frac{\partial}{\partial t} \frac{\partial L}{\partial \dot{u}} = 0 \)

Equations of motion are the same in any inertial frame. Inertial frames are related by Galilean transformation:

\[ x = x' + v_x t \]
\[ t = t' \]