12c) The KAM Theorem

All rational tori are unstable. This is not the case for irrational tori.
Instability occurs for $(\hat{D}_0, \tilde{m}) = \gamma$ for all $\gamma$. So we have to stay away from these frequencies such that

$$\sum_{l,m=0}^{\infty} \frac{k_{\pi}((\hat{D}_0, \tilde{m}))}{2 \pi (a_0, V_0)} e^{i m \tilde{u} \phi_0}$$

Converges for $L \to \infty$

Exclude frequencies with $|m \cdot \tilde{u}| < \gamma$ but we also will to end up with a finite part of phase space.

Consider first the case that $H_0 = \tilde{J}_1 + x \tilde{J}_2$

Set $\gamma = 1$, $\nu_2 = \gamma$

Small denominator relation $m_1 + 2 m_2 = \gamma$

We cannot adjust $\gamma$ to stay away from a small denominator.

This leads to the condition that $k_{\pi}(\hat{D})$ is invertible

$$\text{det } D_x k_{\pi}(\hat{D}) \neq 0 \Rightarrow \text{det } D_x D_{xx} H \neq 0$$

This is called the Hessian condition.

\[ \begin{array}{c|c}
\text{no good} & \text{good} \\
\hline
x & \gamma \\
\end{array} \]