$z \in C(\phi, \eta)$ is a continuous function for $\eta \geq 0$ it maps a circle to a deformed closed curve with the same area.

For larger $\eta$ the angles move faster and for smaller $\eta$ they move slower. We can find a curve such that the $\phi$ values are not changed by the map.

The intersection points are fixed points.

Because the area is preserved, their number is necessarily even.

Let us look at an intersection.