Example: Particle moving on a circle

\[ H = \frac{p_0^2}{2mR^2} \quad H \text{ en} \quad E = \left( \frac{\partial S_0}{\partial \theta} \right)^2 \frac{1}{2mR^2} \]

\[ \Rightarrow \frac{\partial S_0}{\partial \theta} = \sqrt{2mE R^2} \]

\[ S_0(\theta, t) = \sqrt{2mE R^2} \theta \]

\[ \mathcal{D} = \frac{1}{2m} \int_{-\frac{L}{2}}^{\frac{L}{2}} p_x^2 \, dx = \frac{1}{2} \int_{-\frac{L}{2}}^{\frac{L}{2}} p_0 \, d\theta = \sqrt{2mE} R \equiv p_0 \]

\[ \Rightarrow E(\mathcal{D}) = \frac{p_0^2}{2mR^2} \]

\[ S(\theta, \tau) = S(\theta, \omega \tau) = S(\theta, \frac{\tau^2}{2mR^2}) \]

\[ E(\theta) \]

\[ \phi = \frac{\partial S}{\partial \tau} = \theta \]

\( \theta, \tau \) are angle-action variables.