Hamilton-Jacobi theory

\[ H = -\frac{\partial S}{\partial t} \quad \Rightarrow \quad \frac{\partial S}{\partial t} + H(q_i, \frac{\partial S}{\partial q_i}, t) = 0 \]

\[ p_i = \frac{\partial S}{\partial q_i} \]

This is the Hamilton-Jacobi equation for \( S(q_i, t) \)

**Complete integral**: a solution of the HJ eq that contains as many arbitrary constants as independent variables.

**Time + coordinates \( \Rightarrow \) \( S \) + 1 independent variable.**

Since \( S \) is only determined up to a constant, a complete integral is given by

\[ S = F(t, q_1, \ldots, q_s, \dot{q}_1, \ldots, \dot{q}_s) + A \]

Relation between complete integral and solution of the eqs. of motion.

Use \( F \) as generating function for canonical transformation \( q_i, p_i \rightarrow \beta_i, \dot{\beta}_i \)

\[ \beta_i = \frac{\partial F}{\partial q_i}, \quad \dot{\beta}_i = \frac{\partial F}{\partial \dot{q}_i}; \quad H' = H + \frac{\partial F}{\partial t} = 0 \]

\[ \frac{\partial F}{\partial t} = -H \]