Homework Set # 7, due November 5, 2008

1. A spool is made of two uniform disks, each of mass $M$ and radius $R$ joined by a uniform hollow cylinder of radius $r < R$. The disks and cylinder share a common axis which is also their symmetry axis. A thread (massless and thin) is wound around the cylinder and a force $T$ acts at its free end at a fixed angle $\theta$ above the horizontal. The spool can move on a horizontal table.

   a) Find the condition on $T$ for the spool not to move vertically.
   b) Find the moment of inertia of the spool about its axis.
   c) If no slipping occurs find the conditions on $\theta$ that the spool either winds or unwinds.
   d) Find the condition of the coefficient of friction between the spool and the table so that no slippings occurs.

2. You can roll a coin on a table such that it rolls in a circle of radius $R$ and leans inward at an angle $\phi$. Express $\phi$ in terms of the radius $R$ of the circle, the radius $b$ of the coin and the velocity $v$ of the coin. No slipping occurs.

3. An automobile is started from rest with one of its doors open at a right angle. Find the time it takes that the door closes if the acceleration of the car is $f$, the radius of gyration of the door is $r_0$ and its center of mass is at a distance $a$ from its hinges. Hint: Start with the Lagrangian of the system.

4. Consider two particles that can move on a line between two walls. The left particle is connect to the left was by a spring with spring constant $k$, and the right particle to the right wall with a spring with the same constant. The two partitcles are connected to each other by a spring with spring constant $3k$.

   What are the eigenfrequencies and the eigenvectors of this system?