Homework Set # 5, due October 6, 2008

1a. For $V(r) = -\alpha/r$ show that

$$A = \vec{p} \times \vec{L} - m\alpha \vec{r}/r$$

is conserved.

1b. Use this to solve the Kepler problem algebraically (hint: consider $\vec{r} \cdot \vec{A}$).

2. Consider the potential $v(r) = k|\vec{r}|$ felt by mass $m$.

   a) Find the condition for a particle with angular momentum $l$ to move in a stable circular orbit,

   b) Now increase the energy slightly beyond the circular orbit while keeping the angular momentum the same. How much does the angular position change in one period of radial oscillation.

3a. Show that is a particle describe a circular orbit under the influence of an attractive central force directed to a point on the circle, then the force varies as the inverse fifth power of the distance.

3b. Show that for this orbit the total energy of the particle is zero,

3c. Find the period of the motion.

3d. Find $\dot{x}$, $\dot{y}$ and $v$ as a function of the angle around the circle and show that all these quantities are infinite as the particle goes through the center of force.

4. (From last week) Find the orbit $\phi(r)$ for the central potential $U(r) = -\alpha/r + \beta/r^2$ ($\alpha > 0$, $\beta > 0$) for all ranges of energy.