Homework Set # 1, due September 8, 2008

1. Consider a particle that can move freely along a circle that rotates with angular velocity $\Omega$ about the $z$-axis. The axis crosses the center of the circle.

   a) Write down the Lagrangian.

   b) Derive the equations of motion from the Lagrangian and find the equilibrium position of the particle.

   c) What is the condition for stability of the equilibrium position.

2. Let $q_1, \cdots, q_n$ be a set of independent generalized coordinates for a system with $n$ degrees of freedom with Lagrangian $L(q, \dot{q}, t)$. Consider a point transformation to new coordinates $s_1, \cdots, s_n$ given by

   $q_i = q_i(s_1, \cdots, s_n, t), \quad i = 1, \cdots, n. \quad (1)$

   Show that if the Lagrangian is expressed in terms of the $s_k$ and the $\dot{s}_k$ the equations of motion are given by

   $\frac{d}{dt} \frac{\partial L}{\partial \dot{s}_j} = \frac{\partial L}{\partial s_j}. \quad (2)$

3. Two wheels of radius $R$ are at the end of an axle of length $l$. There is no slipping.

   Show that there are two nonholonomic constraints

   $\cos \theta dx + \sin \theta dy = 0, \quad \sin \theta dx - \cos \theta dy = \frac{k}{2}(d\phi + \phi'). \quad (3)$

   where $(x, y)$ is the center of mass. Show that there is also a holonomic constraint.

4. Consider a rod of length $2l$ that can rotate freely in the horizontal plane about a axis attached to its center. The rod is massless and has a mass $m$ on one of its ends, and a string of length $l$ with a mass $m$ attached at its other end. The string makes an angle $\theta$ with the normal of the horizontal plane. Write down the Lagrangian of this system.

   ![Figure 1: Figure for problem 4](image-url)