Action angle variables

\[ S = S_0 - E t \]
\[ S_0 = \sum_k \phi_k (q_k, \theta_k) \]

\[ (q, p) \xrightarrow{\hat{S}(q, \theta)} (\phi, J) \]
\[ p_\alpha = \frac{\partial S_0}{\partial q_\alpha}, \quad q_\alpha = \frac{\partial S_0}{\partial p_\alpha} \]

Solution of HJ equation

\[ (q, p) \rightarrow \mathcal{H} = 0 \]

What is \( S \)?
\[ J_{\alpha} \text{ is constant on } \mathcal{C}_n \]

\[ \int_{\mathcal{C}_n} d\phi_\alpha = 2\pi \]
\[ \int_{\mathcal{C}_n} \frac{d\phi_\alpha}{\partial \alpha} = \int_{\mathcal{C}_n} \frac{d\phi_\alpha}{\partial q_\alpha} \]
\[ \frac{\partial S}{\partial q_\alpha} = \frac{\partial S}{\partial p_\alpha} = \frac{\partial S}{\partial q_\alpha} \]
\[ = \frac{2}{\partial q_\alpha} \int_{\mathcal{C}_n} p_\alpha d\phi_\alpha = 2\pi \]

This suggests that
\[ J_{\alpha} = \frac{1}{2\pi} \int_{\mathcal{C}_n} p_\alpha d\phi_\alpha \]

Assume that we have solved the HJ eqn.

then
\[ J_{\alpha} = \frac{1}{2\pi} \int_{\mathcal{C}_n} \frac{\partial S}{\partial q_\alpha} \]

This gives us
\[ J_{\alpha}(Q) \]