VII(11) Action Angle variables

If motion is on ellipses it makes sense to use $\mathbf{J}_2$ as one variable and an angle along the ellipse as the other variable. $\mathbf{J}_2$ is a constant of motion.

Our construction will be such that $\mathbf{J}_2$ is the momentum corresponding to $\phi_2$

$$\mathbf{J}_2 = -\frac{\partial H}{\partial \phi_2} = 0 \quad \Rightarrow \quad H \text{ does not depend on } \phi_2$$

Angles are normalized as $\int_0^{2\pi} \phi_2 = 2\pi$

$$\phi_2 = \frac{2\pi}{\mathbf{J}_2} \equiv \phi^2 (\tau) \quad \Rightarrow \quad \phi_2 = \omega^2 \tau + \Theta^2$$

$\Rightarrow$ motion is on tori

What is the canonical transformation from $(\mathbf{r}, \mathbf{p}) \rightarrow (\phi, \mathbf{J})$?

We want it to be canonical so that $\phi, \mathbf{J}$ satisfy Hamilton's eqs. of motion.