Poincaré section

System with 2 dof
\( x_1, x_2, \dot{x}_1, \dot{x}_2 \)
Conserved energy \( E \)
\( \Rightarrow \) trajectory in \( 3d \) space

Only points with \( \dot{x}_1 > 0 \)

Poincaré section: section of a trajectory with the \( x_2 - \dot{x}_2 \) plane

two conditions on four variables: \( E, x_1 = 0 \)
\( \Rightarrow \) points cover a \( 2d \) area

If we have two conserved quantities:
three conditions on four variables \( \Rightarrow \)
points are on a curve

Example h.o. in 2 \( d \)
two conserved quantities
\[
\frac{1}{2} \dot{x}_1^2 + \omega_1^2 x_1^2 = E_1
\]
\[
\frac{1}{2} \dot{x}_2^2 + \omega_2^2 x_2^2 = E_2
\]