

$F$  is called the generating function

$$dF = \sum p_i dq_i - \sum P_i dQ_i + (H' - H) dt$$

$$\Rightarrow p_i = \frac{\partial F}{\partial q_i} \quad P_i = -\frac{\partial F}{\partial Q_i} \quad H' = H + \frac{\partial F}{\partial t}$$

Example:  $F = \sum q_i Q_i$

$$\Rightarrow p_i = Q_i \quad P_i = -q_i$$

momenta and coordinates are interchanged

alternative transformations can be obtained by Legendre transformations

$$\phi = F + \sum P_i Q_i$$

$$\Rightarrow d\phi = \cancel{-P_i dQ_i} + p_i dq_i + H' dt - H dt + \cancel{P_i dQ_i} + dP_i Q_i$$

$$\Rightarrow p_i = \frac{\partial \phi}{\partial q_i} \quad Q_i = \frac{\partial \phi}{\partial P_i} \quad H' = H + \frac{\partial \phi}{\partial t}$$

$$\phi = \phi(q, P, t)$$