

$$\Rightarrow S_0 = \int (2(E-u) \sum a_i u_i da_i)^{\frac{1}{2}}$$

Free particle
in potential

$$T = \frac{1}{2} \left(\frac{dx}{dt} \right)^2$$

$$\text{then } S_0 = \int \sqrt{2m(E-u)} \sqrt{\frac{(dx)^2}{dt^2}} \int dt$$

$$\text{Jacobi principle } \delta S_0 = \delta \int \sqrt{2m(E-u)} dr = 0$$

$$\text{free particle } \Rightarrow u=0 \Rightarrow \delta \int dr = 0$$

\Rightarrow particle moves according to the shortest path between two points (straight line)
"geodesic motion".