Poisson theorem

$$\frac{df}{dt}=0 \quad \frac{dg}{dt}=0 \quad \Rightarrow \quad \frac{d}{dt}[f,g]=0$$

Use Jacobi identity with $h=H$

$$[H,[f,g]]+[f,\underbrace{[g,H]}_{=0}]+[g,\underbrace{[H,f]}_{=0}]=0$$

$$\Rightarrow \frac{d}{dt}[f,g]=0 \qquad \text{no explicit time dependence}$$

VII c) Hamilton's principle

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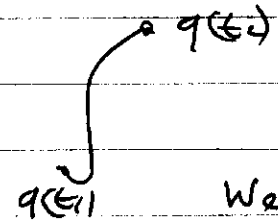

$$S=\int_{t_1}^{t_2} L\,dt$$

We consider the action as a function of the endpoint for fixed initial point

Then
$$\delta S=\int_{t_1}^{t_2}\left(\frac{\partial L}{\partial q}\delta q+\frac{\partial L}{\partial \dot q}\delta \dot q\right)dt$$

$$=\int_{t_1}^{t_2}\Big[\underbrace{\frac{\partial L}{\partial q}\delta q-\frac{d}{dt}\frac{\partial L}{\partial \dot q}\delta q}_{=0 \text{ by EL}}+\frac{d}{dt}\left(\frac{\partial L}{\partial \dot q}\delta q\right)\Big]dt$$

$$=-\frac{\partial L}{\partial \dot q}\Big|_{t_1}\underbrace{\delta q(t_1)}_{=0}+\underbrace{\frac{\partial L}{\partial \dot q}\Big|_{t_2}}_{=p(t_2)}\delta q(t_2)$$

$$\Rightarrow \frac{\delta S}{\delta q(t_2)}=p(t_2)$$

We can also consider S as an explicit function of time by taking $t_2=t$ at fixed t_1