If the kinetic energy is quadratic in the velocities then

\[ L = \frac{1}{2} \sum g_{ij} \dot{q}_i \dot{q}_j - U = p_i \frac{\partial L}{\partial \dot{q}_i} = g_{ij} \ddot{q}_j \]

\[ H = p_i \dot{q}_i - L = \dot{q}_i g_{ij} \ddot{q}_j - \frac{1}{2} g_{ij} \dot{q}_i \dot{q}_j + U \]

\[ = T + U \]

Sometimes it is useful to transform only part of the coordinates.

\[ i.e. \quad q, \dot{q}, \dot{q}, \ddot{q} \rightarrow q, p, \ddot{q}, \dddot{q} \]

this leads to the so-called Routhian.

\[ \text{VII 4) Poisson brackets \& SY3} \]

Let us consider a function on phase space

\[ f(q, p, t) \]

the

\[ \frac{\partial f}{\partial t} = \frac{\partial f}{\partial q} \dot{q} + \frac{\partial f}{\partial p} \dot{p} + \frac{\partial f}{\partial x} x \cdot \frac{\partial H}{\partial p_x} - \frac{\partial H}{\partial q_x} \]

\[ = \frac{\partial f}{\partial t} = \frac{\partial f}{\partial q} \dot{q} + [H, f] \]

\[ [H, f] = \sum \left( \frac{\partial H}{\partial p_x} \frac{\partial f}{\partial q} - \frac{\partial H}{\partial q_x} \frac{\partial f}{\partial p} \right) \]

Poisson bracket

classical limit of commutator