(VI γ)

# The free asymmetric top § 37

$I_3 > I_2 > I_1$

Free $\Rightarrow$ $E, M$ are conserved

$$I_1 \Omega_1^2 + I_2 \Omega_2^2 + I_3 \Omega_3^2 = 2E$$
$$I_1^2 \Omega_1^2 + I_2 \Omega_2^2 + I_3^2 \Omega_3^2 = M^2$$

6 dof
4 conserved quantities } can be integrated
we do not solve the equations but study motion in $\vec{M}$ space

The equation can also be written as

$$M_1^2 + M_2^2 + M_3^2 = M^2 \qquad \text{sphere}$$
$$\frac{M_1^2}{I_1} + \frac{M_2^2}{I_2} + \frac{M_3^2}{I_3} = 2E \qquad \text{ellipsoid}$$

endpoint of $\vec{M}$ moves on the intersection of the two

a) $2EI_1 < \vec{M}^2 < 2EI_3$ : ellips always intersects sphere

$\vec{M}^2 \to 2EI_1$ then $M_2 \to 0$, $M_3 \to 0$

then $M_1^2 + M_2^2 \frac{I_1}{I_2} + M_3^2 \frac{I_1}{I_3} = 2EI_1$

$$M_1^2 + M_2^2 + M_3^2 = \vec{M}^2$$
$$\Rightarrow \quad M_2^2 \underbrace{\left(1 - \frac{I_1}{I_2}\right)}_{>0} + M_3^2 \underbrace{\left(1 - \frac{I_1}{I_3}\right)}_{>0} = \vec{M}^2 - 2EI_1 > 0$$

equation for ellips
stable