

(63)

$$\Rightarrow \Omega_1 = \dot{\theta} \cos \psi + \dot{\phi} \sin \theta \sin \psi$$

$$\Omega_2 = -\dot{\theta} \sin \psi + \dot{\phi} \sin \theta \cos \psi$$

$$\Omega_3 = \dot{\psi} + \dot{\phi} \cos \theta$$

Example: <sup>Free</sup> Symmetric top:  $I_1 = I_2$

$$T_{rot} = \frac{1}{2} I_1 (\Omega_1^2 + \Omega_2^2) + \frac{1}{2} I_3 \Omega_3^2$$

$$= \frac{1}{2} I_1 (\dot{\phi}^2 \sin^2 \theta + \dot{\theta}^2) + \frac{1}{2} I_3 (\dot{\psi} + \dot{\phi} \cos \theta)^2$$

$x_1, x_2$  axis is arbitrary; we can choose  $\psi = 0$   
and  $x_3$  is chosen along principal axis

No torques  $\Rightarrow \frac{d\vec{P}}{dt} = 0 \quad \vec{P} = (I_1 \Omega_1, I_1 \Omega_2, I_3 \Omega_3)$   
 $\Rightarrow \frac{d\Omega_1}{dt} = 0$

$$M_1 = I_1 \Omega_1 = I_1 \dot{\theta}$$

$$M_2 = I_2 \Omega_2 = I_1 \dot{\phi} \sin \theta$$

$$M_3 = I_3 \Omega_3 = I_3 (\dot{\phi} \cos \theta + \dot{\psi})$$

choose  $z$ -axis  $\parallel \vec{P}$

$$\Rightarrow \dot{\theta} = 0 \Rightarrow M_1 = 0$$

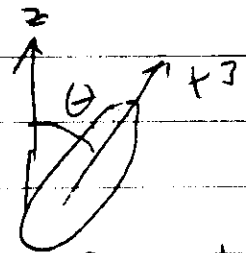
$$\Rightarrow \Omega_2 = \dot{\phi} \sin \theta$$

$$M_2 = I_1 \dot{\phi} \sin \theta$$

$$M_3 = I_3 (\dot{\phi} \cos \theta + \dot{\psi})$$

$$\Rightarrow \dot{\phi} = \frac{\dot{P}}{I_1} \quad \text{precession}$$

$$I_3 \Omega_3 = M \cos \theta$$



$x_3 \parallel$  lines of nodes

$$\Rightarrow \psi = 0$$

$$\Rightarrow x_1 \perp z$$

$$\Rightarrow M_1 = 0$$

$$\Rightarrow \vec{P} \text{ in } x_1 x_3 \text{ plane}$$

$$M_3 = P \cos \theta$$

$$M_2 = P \sin \theta$$