Rotational equations

\[ \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\phi}_i} \right) - \frac{\partial L}{\partial \phi_i} = \tau_i \]

\[ \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\phi}_i} \right) - \frac{\partial L}{\partial \phi_i} = -\frac{\partial \mu}{\partial \phi_i} \]

\[ \Rightarrow \frac{d}{dt} M_i = -\frac{\partial \mu}{\partial \phi_i} \]

Change of potential energy by infinitesimal rotation

\[ \delta \mu = - \sum_k f_k \delta \phi \]

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Torque \( \tau = \sum_k f_k \)

\[ \tau_i = - \frac{\partial \mu}{\partial \phi_i} = \frac{d}{dt} M_i = \tau_i \]

\[ \tau = \tau_1 + \tau_2 \]

The \( \tau = \tau_1 + \tau_2 + \tau_3 = \sum_k f_k \)

If \( \tau = 0 \) then torque is independent of the choice of the origin eg \( \tau = \tau_1 + \tau_2 + \tau_3 \)

If \( \tau \perp F \) then \( \tau = F \times \dot{\phi} \)

\[ \Rightarrow \tau = \dot{\phi} \times F \]