I.4) Galileo's relativity principle

It is always possible to find coordinates such that time and space are homogeneous and space is isotropic. Such frame is called an inertial frame.
This means that a free particle has the same equations of motion all over space and time.

$\Rightarrow$ Lagrangian of a free particle does not depend on q_k and t. It depends only on

$$v^2 = \sum \dot{q}_k^2$$

$$\Rightarrow L_{free} = L(v^2)$$

Lagrange equation: $\frac{d}{dt}\frac{\partial L}{\partial v} = 0$

$$\Rightarrow \frac{\partial L}{\partial v} = \text{constant} \neq v = \text{constant}$$

Law of inertia: $\vec{v}$ is constant for free motion.

Equations of motion are the same in any inertial frame. Different inertial frames are related by Galileo transformation:

$$r = r' + vt$$
$$t = t'$$