transformation of inertia tensor under $\vec{r} \to \vec{r} + a$

$$\delta_{\ell p} \vec{x}^2 - x_\ell x_r \to \delta_{\ell p} \vec{x}^2 - x_\ell x_r$$
$$+ 2x.a\, \delta_{\ell p} + \delta_{\ell p} \vec{a}^2 - x_\ell a_p - x_p a_\ell$$
$$- a_\ell a_p$$

$$\int \mu(\vec{r})\, d^3\vec{r}\ x_k = 0$$

$$\Rightarrow \quad I_{\ell p} \to I_{\ell p} + \delta_{\ell p} \vec{a}^2 - a_\ell a_p$$

VI f) Angular momentum

$$\vec{J} = \int d^3x\ \mu(x)\ \vec{x} \times \underbrace{\vec{v}}_{= \vec{\omega} \times \vec{x}}$$

$$J_k = \int d^3x\ \mu(x)\ \underset{\underset{-\varepsilon_{jik}}{\shortparallel}}{\varepsilon_{ijk}}\ x_i\ \varepsilon_{jmn}\, \omega_m x_n \qquad \varepsilon_{jik}\varepsilon_{jmn} = \delta_{mi}\delta_{kn} - \delta_{in}\delta_{mk}$$

$$= \int d^3x\ \mu(x)\ (\delta_{in}\delta_{mk} - \delta_{mi}\delta_{kn})\ x_i\, \omega_m x_n$$
$$= \int d^3x\ \mu(x) (x^2 \omega_k - (x.\omega)\, x_k)$$
$$= \int d^3x\ \mu(x) (x^2 \delta_{km} - x_m x_k)\, \omega_m$$

$$= I_{km}\, \omega_m \qquad \boxed{J = I\omega}$$

principal axis: $J_k = I_k \Omega_k$