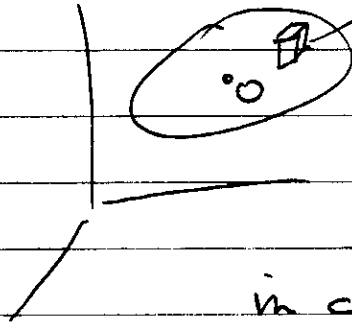
VI e Kinetic Energy

$\mu(x)$ is mass density



$$T = \frac{1}{2}\int \dot{x}^2(r)\,\mu(\vec{r})\,d^3 r$$

in cm frame $\dot{\vec{x}} = \vec{\omega} \times \vec{x}$

$$\Rightarrow \dot{\vec{x}}^2 = (\vec{\omega} \times x)\cdot(\vec{\omega} \times x)$$
$$= \omega^2 x^2 - (\omega \cdot x)^2$$

$$\Rightarrow T = \frac{1}{2}\omega_\ell \omega_p \underbrace{\int d^2 r\, \mu(\vec{r})\,(\delta_{\ell p} x^2 - x_\ell x_p)}_{I_{\ell p}}$$

↑ moment of inertia tensor

Total Kinetic energy

$$T = \frac{1}{2} M V^2 + \frac{1}{2}\sum_{\ell p} \omega_\ell I_{\ell p} \omega_p$$

Note that if we calcute $\frac{1}{2}\sum m\,(\vec{V} + \vec{\Omega}\times\vec{r})^2$
then $\sum m \vec{V} \cdot \vec{\Omega} \times \vec{r} = 0$

$= \sum m \vec{r} \cdot \vec{V} \times \vec{\Omega}$

$= 0$ because $\vec{r}$ is w.r.t. cm

$I_{\ell p} = I_{p\ell} \Rightarrow I$ can be diagonalized

$I_{\ell p}\,\omega_\ell \omega_p >$ because $T > 0$