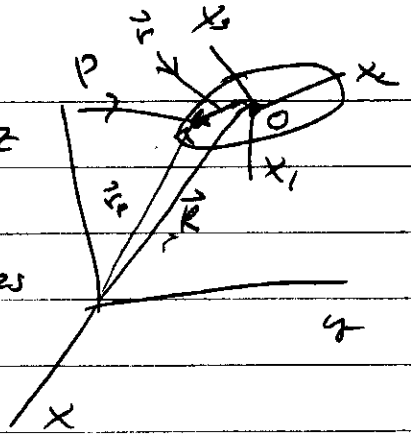
VIId Rigid body motion



XYZ inertial frame

$X_1 X_2 X_3$ body fixed coordinates (origin in c.m.)

orientation of X_1, X_2, X_3 : 3 angles

$\vec{R}$ 3 coordinates

$\Rightarrow$ rigid body has 6 dof.

cm displacement

relative to cm

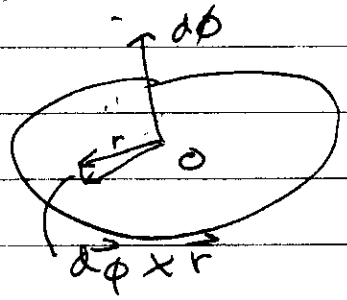
motion of point P: $d\vec{r}_p = d\vec{R} + \underbrace{d\vec{\phi} \times \vec{r}}$

divide by dt

$\Rightarrow \vec{v}_p = \vec{V} + \vec{\Omega} \times \vec{r}$

$\vec{V}$: cm velocity

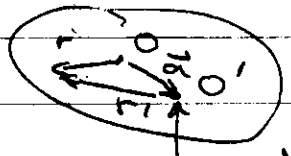
$\vec{\Omega} = \dfrac{d\vec{\phi}}{dt}$



- valid for any choice of O

Let us choose different coordinates O'

with $\vec{r} = \vec{r}' + \vec{a}$



$\Rightarrow v_p = \vec{V} + \vec{\Omega} \times \vec{r}' + \vec{\Omega} \times \vec{a}$

$\Rightarrow \vec{V}' = \vec{V} + \vec{\Omega} \times \vec{a}$ $\quad \vec{\Omega}' = \vec{\Omega}$

velocity of O' is V'

$\uparrow$ angular velocity for O'

if $\vec{V} \perp \vec{\Omega} \Rightarrow \vec{v}_p \perp \Omega$

$\Rightarrow V' \perp \Omega$

it is possible to find $\vec{a}$ such that $V' = 0$

then there is an instantaneous axis of rotation through O'