Let $r = \sqrt{x^2+y^2}$

$x = r\cos\alpha$ $\qquad$ $x' = r\cos(\alpha+\varphi)$

$y = r\sin\alpha$ $\qquad$ $y' = r\sin(\alpha+\varphi)$

$$\Rightarrow \quad x' = r[\cos\alpha\cos\varphi - \sin\alpha\sin\varphi]$$
$$y' = r[\sin\alpha\cos\varphi + \sin\varphi\cos\alpha]$$

$$\Rightarrow \quad x' = x\cos\varphi - \sin\varphi\, y$$
$$y' = y\cos\alpha + x\sin\alpha$$

$$\Rightarrow \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$O(2)$ rotations are matrices of the form $\begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix}$

This set of matrices is a Lie group (it is continuous). Then a rotation by φ can be viewed as n successive rotations by φ/n.

$$O(\varphi) = \Big[\, O\big(\underset{\underset{\varepsilon}{\parallel}}{\tfrac{\varphi}{n}}\big) \,\Big]^n$$

$$O(\varepsilon) = \begin{pmatrix} \cos\varepsilon & -\sin\varepsilon \\ \sin\varepsilon & \cos\varepsilon \end{pmatrix} = 1 + \varepsilon \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

infinitesimal generator

Lie groups are determined by the properties of the infinitesimal generators.